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Fuzzification of Simpson's 1/3 Rule and Development of its Computer Program

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Abstract

Many applications of integral calculus can be found in a variety of disciplines, such as engineering, statistics, finance, actuarial science, etc. The evaluation of expressions involving these integrals can occasionally get extremely challenging. As a result, numerous numerical methods (such as numerical integration) have been created to make the integral simpler. For instance, recent years have seen a focus on Bayesian and empirical Bayesian methods in statistics. Numerical integration is used to approximate numerical values that cannot be integrated analytically. Different numerical integration methods e.g. Newton-Cotes, Romberg integration, Gauss Quadrature and Monte Carlo integration are used to assess those functions that can't be integrated systematically. Newton-Cotes methods have been used to interpolate polynomials. One of the Newton-Cotes methods does not have any restriction on segmentation. But, there must be an even number of segments for the Simpson 1/3 rule. In this study an attempt has been made to fuzzify the Simpson 1⁄3 rule and developed computer programs for the same. Also, a comparison between the classical and fuzzified Simpson's 1/3 rule has been done.

1. Brief Introduction:

Integration, according to Kaw and Keteltas, is a technique for calculating the area under a function that has been graphed. Several professions, including engineering, statistics, finance, actuarial science, and others, use integral calculus extensively. The evaluation of expressions involving these integrals can occasionally get extremely challenging. As a result, numerous numerical methods (such as numerical integration) have been created to make the integral simpler. For instance, recent years have seen a focus on Bayesian and empirical Bayesian methods in statistics. Numerical integration is used to approximate numerical values that cannot be integrated analytically. Different numerical integration methods e.g. Newton-Cotes, Romberg integration, Gauss Quadrature and Monte Carlo integration are used to assess such functions that cannot be integrated systematically. Newton-Cotes methods interpolate polynomials. One of the Newton-Cotes methods does not have any restriction on segmentation. But in Simpson's 1⁄3 rule, the number of subdivision is multiple of 2, and in Simpson's 3/8 rule, the number of subdivision is a multiple of 3. In the Boole's rule, the number of subdivision is multiple of 4 whereas in the Weddle's rule, this number of subdivision is multiple of 6. Mettle F.O. et. al developed a new Trapezium rule and the numerical integration methods having no restriction on the number of segmentation[1].

Integrals are significant part of mathematical analysis. Integrals are not only effective in mathematics analysis, but in other field also. However many functions are not possible to calculate precisely, that means some functions cannot be calculated with analytical mathematical methods. Due to these grounds another method was established known as numerical integration with the intention to find approximate result of required integral. As per requirement one can use numerical methods, which allows to calculate the result with specific error. There are a few basic methods of numerical integration such as Trapezoidal rule, Simpson's rule etc. In all the techniques approximated value of integral is calculated, but with various errors[2].

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Charles A. Thomson conducted research on numerical integration techniques for use in companion circuit method of transient circuit analysis. Numerical integration methods used in circuit transient analysis packages may not be the most accurate approximations of the actual circuit response. This study focuses on these numerical integration techniques and their inaccuracy[3].

Li J. made a study on composite trapezoidal rule for the estimation of Cauchy principal value integral on circle. This paper carried out a research on the convergence rate of the trapezoidal rule. It has made a conclusion that the super convergence rate of the composite trapezoidal rule occurs at middle of each subinterval[4].

Zhao W. And Zhang Z. Conducted a studyon Trapeziodal Rule based on derivatives for Riemann Stieltjes. This discussion focuses on the derivative based trapezoidal rule for Riemann Stieltjes integral and also investigated the accuracy rate for this[5].

Moheuddin et al. compared three numerical integration methods namely Trapezoidal rule, Simpson's 1/3 rule and Simpson's 3/8 rule. In this study the rate of precision of these methods has been compared using error values of these methods. Graphical representations have also been made to verify the results. Simpson's 1/3 rule has been found to be the most efficient one among these three techniques[6].

In numerical analysis, Simpson's 1/3 Rule is a technique for approximating the definite integral.

This study is mainly dealing with Fuzzification of Simpson's 1/3 rule and its computer application and comparison of Classical Simpson's Rule with the Fuzzified Simpson's 1/3 Rule.

2. Fuzzification of Simpson's 1/3 Rule:

Fuzzification is the conversion of crisp items into fuzzy items. Therefore to fuzzify the classical Simpson's 1/3 rule, the values of this technique have been replaced by triangular fuzzy number. As result of this, the following expression is derived for fuzzified Simpson's 1/3 rule.

Let $Y = F(X)$ be a function.

$$\int_{X_0}^{X_n} F(X)\,dX = \frac{H}{[3,3,3]}\left[\begin{array}{c}(sum\ of\ the\ extreme\ ordinates) + \\ [2,2,2] \times (sum\ of\ the\ even\ ordinates) + [4,4,4] \times (sum\ of\ the\ odd\ ordintaes)\end{array}\right]$$

$$= \frac{H}{[3,3,3]}[(Y_0 + Y_n) + [2,2,2](Y_2 + Y_4 + \cdots) + [4,4,4](Y_1 + Y_3 + \cdots)]$$

where $X_0 = [X_0', X_0'', X_0''']$, $X_n = [X_n', X_n'', X_n''']$ are triangular fuzzy numbers and Y_0, Y_n are the first and the last ordinates and $Y_1, Y_3, \dots$ are remaining odd ordinates in fuzzy form i.e. $Y_1 = [Y_1', Y_1'', Y_1''']$, $Y_3 = [Y_3', Y_3'', 3]$, … , $Y_{n-1} = [Y_{n-1}', Y_{n-1}'', Y_{n-1}''']$ and $Y_2, Y_4, \dots Y_{n-2}$ are even ordinates in triangular fuzzy number form i.e. $Y_2 = [Y_2', Y_2'', Y_2''']$, $Y_4 = [Y_4', Y_4'', Y_4''']$, … , $Y_{n-2} = [Y_{n-2}', Y_{n-2}'', Y_{n-2}''']$

The f.m.f. of $Y_0, Y_1, Y_2, Y_3, Y_4, \dots, Y_n$ are respectively,

$$\mu_{Y_0}(X) = \begin{cases} \frac{X - Y_0'}{Y_0'' - Y_0'} & where\ Y_0' \le X \le Y_0'' \\ \frac{X - Y_0'''}{Y_0'' - Y_0'''} & where\ Y_0'' \le X \le Y_0''' \\ 0 & otherwise \end{cases}$$

$$\mu_{Y_1}(X) = \begin{cases} \frac{X - Y_1'}{Y_1'' - Y_1'} & where\ Y_1' \le X \le Y_1'' \\ \frac{X - Y_1'''}{Y_1'' - Y_1'''} & where\ Y_1'' \le X \le Y_1''' \\ 0 & otherwise \end{cases} \dots\dots$$

$$\mu_{Y_n}(X) = \begin{cases} \frac{X - Y_n'}{Y_n'' - Y_n'} & where\ Y_n' \le X \le Y_n'' \\ \frac{X - Y_n'''}{Y_n'' - Y_n'''} & where\ Y_n'' \le X \le Y_n''' \\ 0 & otherwise \end{cases}$$

and α cut is $[Y_n]^\alpha = [Y_n' + (Y_n'' - Y_n')\alpha, Y_n''' - (Y_n''' - Y_n'')\alpha]$

Similarly the f.m.f. of $X_0, X_1, \dots, X_n$ are respectively

$$\mu_{X_0}(X) = \begin{cases} \frac{X - X_0'}{X_0'' - X_0'} & where\ X_0' \le X \le X_0'' \\ \frac{X - X_0'''}{X_0'' - X_0'''} & where\ X_0'' \le X \le X_0''' \\ 0 & otherwise \end{cases}$$

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$$\mu_{X_1}(X) = \begin{cases} \frac{X-X_1'}{X_1''-X_1'} & where\ X_1' \le X \le X_1'' \\ \frac{X-X_1'''}{X_1''-X_1'''} & where\ X_1'' \le X \le X_1''' \\ 0 & otherwise \end{cases} \ldots\ldots$$

$$.\mu_{X_n}(X) = \begin{cases} \frac{X-X_n'}{X_n''-X_n'} & where\ X_n' \le X \le X_n'' \\ \frac{X-X_n'''}{X_n''-X_n'''} & where\ X_n'' \le X \le X_n''' \\ 0 & otherwise \end{cases}$$

and α cut is $[X_n]^\alpha = [X_n' + (X_n'' - X_n')\alpha, X_n''' - (X_n''' - X_n'')\alpha]$

Example 1 Let us evaluate $\int_{[-.01,0,.01]}^{[3.99,4,4.01]} e^X dX$

Here F(X)= e^X and H=[0.99,1,1.01]

The f.m.f. of H is

$$\mu_H(X) = \begin{cases} \frac{X-0.99}{1-0.99} & where\ 0.99 \le X \le 1 \\ \frac{X-1.01}{1-1.01} & where\ 1 \le X \le 1.01 \\ 0 & otherwise \end{cases}$$

Let us form a table for the X and Y values

X	Y=F(X)
X_0= -0.01 0 0.01	Y_0= 0.99005 1 1.01005
X_1= 0.98 1 1.02	Y_1= 2.66446 2.71828 2.77319
X_2= 1.97 2 2.03	Y_2= 7.17068 7.38906 7.61409
X_3= 2.96 3 3.04	Y_3= 19.298 20.0855 20.9052
X_4= 3.95 4 4.05	Y_4= 51.9354 54.5981 57.3975

By fuzzified Simpson's 1/3 rule

$$\int_{X_0}^{X_n} F(X)\,dX = \frac{H}{[3,3,3]}\left[\begin{array}{c}(sum\ of\ the\ extreme\ ordinates) + \\ [2,2,2] \times (sum\ of\ the\ even\ ordinates) + [4,4,4] \times (sum\ of\ the\ odd\ ordintaes)\end{array}\right]$$

$$= \frac{H}{[3,3,3]}[(Y_0 + Y_4) + [2,2,2] \times (Y_2) + [4,4,4] \times (Y_1 + Y_3)]$$

Say Y = [51.1884, 53.8557, 56.6776]

The f.m.f. of Y is

$$\mu_Y(X) = \begin{cases} \frac{X-51.1884}{53.8557-51.1884} & where\ 51.1884 \le X \le 53.8557 \\ \frac{X-56.6776}{53.8557-56.6776} & where\ 53.8557 \le X \le 56.6776 \\ 0 & otherwise \end{cases}$$

and α cut is $[Y]^\alpha = [51.1884 + (53.8557 - 51.1884)\alpha, 56.6776 - (56.6776 - 53.8557)\alpha]$

Example 2 Let us evaluate $\int_{[1.99,2,2.01]}^{[9.99,10,10.01]} \frac{dX}{[1,1,1]+X}$

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Here F(X)= $\int_{[1.99,2,2.01]}^{[9.99,10,10.01]} \frac{dX}{[1,1,1]+X}$ and H=[0.99,1,1.01]

The f.m.f. of H is

$$\mu_H(X) = \begin{Bmatrix} \frac{X-0.99}{1-0.99} & where\ 0.99 \le X \le 1 \\ \frac{X-1.01}{1-1.01} & where\ 1 \le X \le 1.01 \\ 0 & otherwise \end{Bmatrix}$$

Let us form the table for X and Y

X	Y=F(X)
X_0= 1.99 2 2.01	Y_0= 0.332226 0.333333 0.334448
X_1= 2.98 3 3.02	Y_1= 0.248756 0.25 0.251256
X_2= 3.97 4 4.03	Y_2= 0.198807 0.2 0.201207
X_3= 4.96 5 5.04	Y_3= 0.165563 0.166667 0.167785
X_4= 5.95 6 6.05	Y_4= 0.141844 0.142857 0.143885
X_5= 6.94 7 7.06	Y_5= 0.124069 0.125 0.125945
X_6= 7.93 8 8.07	Y_6= 0.110254 0.111111 0.111982
X_7= 8.92 9 9.08	Y_7= 0.0992063 0.1 0.100806
X_8= 9.91 10 10.09	Y_8= 0.0901713 0.0909091 0.091659

By fuzzified Simpson's 1/3 rule

$$\int_{X_0}^{X_n} F(X)\,dX = \frac{H}{[3,3,3]}\begin{bmatrix} (sum\ of\ the\ extreme\ ordinates) + \\ [2,2,2] \times (sum\ of\ the\ even\ ordinates) + \ [4,4,4] \times (sum\ of\ the\ odd\ ordintaes) \end{bmatrix}$$

$$= \frac{H}{[3,3,3]}[(Y_0 + Y_8) + [2,2,2] \times (Y_2 + Y_4 + Y_6) + [4,4,4] \times (Y_1 + Y_3 + Y_5 + Y_7)]$$

Say Y=[1.27861, 1.29951, 1.32089]

The f.m.f. of Y is

$$\mu_Y(X) = \begin{Bmatrix} \frac{X-1.27861}{1.29951-1.27861} & where\ 1.27861 \le X \le 1.29951 \\ \frac{X-1.32089}{1.29951-1.32089} & where\ 1.29951 \le X \le 1.32089 \\ 0 & otherwise \end{Bmatrix}$$

and α cut is $[Y]^\alpha = [1.27861 + (1.29951 - 1.27861)\alpha, 1.32089 - (1.32089 - 1.29951)\alpha]$

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3. Fuzzified Simpson's 1/3 Rule Vs Classical Fuzzified Simpson's 1/3 Rule:

Fifteen examples have been considered to compare the fuzzified Simpson 1/3 rule with the classical Simpson 1/3 rule. For both the methods, C++ programs have been developed. The solutions of these examples are calculated by using these programs. The outputs have been presented in the Table 1

Table -1 Output of the C++ Program Developed for the Fuzzified Simpson's 1/3 Rule and Classical Simpson's 1/3 Rule

Sl No	Crisp root	Fuzzy Triangular Number	Defuzzified value
1	6.387	6.04052, 6.3911, 6.76031	6.39121
2	0.83567	.784495, 0.8357, .888151	.835785
3	0.37578	.343622, 0.37589, .411198	.375996
4	.74685	.709664, 0.74678, .784528	.746855
5	53.8628	51.1884, 53.8557, 56.6776	53.8638
6	1.82781	1.72676, 1.8277,1.92985	1.82785
7	0.79528	0.727494, 0.79528, 0.865996	0.795298
8	1.2986	1.27861, 1.29951, 1.32089	1.29962
9	0.69326	0.653547, 0.69324, 0.734472	0.693254
10	0.40251	0.389019, 0.40245, 0.416413	0.402521
11	0.21451	0.177027, 0.21455, 0.257957	0.214608
12	0.14508	0.116905, 0.14508, 0.176931	0.145096
13	0.78165	0.761544, 0.78165, 0.930776	0.781752
14	0.5203	0.4512, 0.5201, 0.5940	0.5202
15	0.6668	0.642101, 0.6658, 0.689548	0.666932

Comparison of Solutions obtained from Fuzzified Simpson's 1/3 Rule and Classical Simpson's 1/3 Rule:

Appropriate statistical test is used to check whether the results obtained by fuzzified Simpson's 1/3 Rule are different with the results obtained by the classical method.

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The normality of the results is tested by using box-plot and K-S test in order to select a proper statistical test. The box-plot is presented in Figure 1.

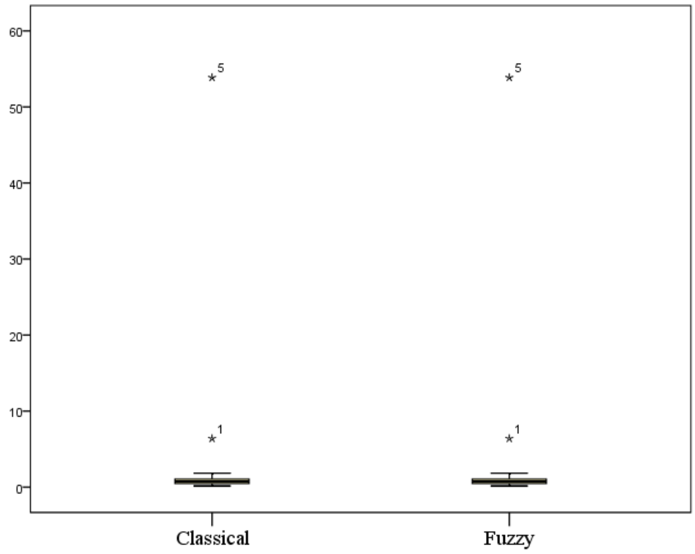

Figure 1: Box-plot of Simpson's 1/3 Rule

From the above box plot it can be concluded that it is less likely that the results follow normal distribution and to confirm this, K-S test has been used. The results of the K-S test are displayed in Table 2.

Table – 2 Results of K-S test

	Kolmogorov-Smirnov		
	Statistic	d.f.	p-value
Classical	.448	15	0.0000000050022
Fuzzy	.448	15	0.0000000050023

From Table-2, it can be observed that the results do not follow normal distribution since p-value<0.01.

Now, Wilcoxon-signed rank test (non-parametric statistical test) is used to compare whether the results obtained by both the methods are statistically significant. The results are shown in Table 3.

Table-3 Findings of Descriptive Statistics and Wilcoxon-signed rank test

	Classical	Fuzzy	z-value	p-value
Mean	4.63739	4.63739		
Median	0.74686	0.74686		

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Std. Deviation	13.70266	13.70264		
Minimum	0.14510	0.14510		
Maximum	53.86386	53.86380	0.437	0.755

From the outcome of Wilcoxon-signed rank test it can be seen that the results are not statistically significant since p-value<0.05. So it can be observed that the solutions of the mathematical problem are more or less same. Finally from this study it has been seen that the Simpson's 1/3 rule in its fuzzified as well as classical form gives the same results.

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